Alexandria Engineering Journal (2014) 53, 669–675

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Alexandria Engineering Journal

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ORIGINAL ARTICLE

A generalized mathematical model to determine the turning movement counts at roundabouts

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Received 29 March 2014; revised 20 May 2014; accepted 4 June 2014
Available online 23 July 2014

KEYWORDS

Roundabouts;
Turning movements;
Traffic counts;
Tracking problem;
Mathematical models

Abstract Traffic turning movement counts at roundabouts is one of the key inputs required for roundabout assessment, control and management. Traditionally, a direct counting is conducted to track a vehicle from entering through circulation until exiting. This counting may be difficult and costly due to the size of roundabout, the vision obstacles, and the continuous traffic flow. Many researchers tried to avoid the tracking problem by counting only at entries and exits, then estimating the movements based on historical data which unfortunately affect the results. Other researchers reduced the tracking problem by counting some turning movements in addition to at entries and exits, then calculating mathematically the remaining movements. This approach is practical and accurate; however, it was applied on limited cases. In this paper, a generalized mathematical model was developed to calculate the most difficult movements based on the easiest movements determined based on the size of monitoring area. The developed model can be used to calculate the turning movements, including the u-turns, for roundabouts with any number of legs. The developed model was presented in O–D matrix forms to be practical and user-friendly. The model was validated against reference count data and the results were found to be satisfactory.

1. Introduction

Roundabouts are one of the widely used types of intersections nowadays. They serve a major role in regulating vehicle turning movements from one direction to another in a safe and efficient manner. Determination of traffic turning movement counts at roundabouts is one of the key inputs required for a variety of traffic analysis, including intersection geometric design, traffic control devices design, traffic impact assessment, capacity estimation, safety evaluation, etc. Standard intersections are relatively straightforward to count, but for roundabouts the situation is different [1]. It may be difficult and costly to track a vehicle from entry through circulation until exit the roundabout due to the size of roundabout and the vision obstacles. Additionally, the continuous entry of new vehicles from all legs makes it difficult to track every vehicle. For these reasons, the traditional and direct counting survey conducted by manual manner, counters, or even video cameras may not necessarily

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2014.06.012
1110-0168

be appropriate as a workable methods especially for large roundabouts and roundabouts with site limitations.

Estimating turning movement volumes at roundabouts, given some level of field measurements as input, is an alternative to full and direct field observations. Based on this approach, several attempts to estimate turning movement volumes were developed making the use of traffic counts only at entries and exits and historical data for turning movement volumes [2–10]. Unfortunately, the results obtained by such methods may be inaccurate due to the influence of the used seed data.

Another approach was developed to reduce the tracking problem by counting some turning movements in addition to in-flows and out-flows. Then, it could be possible to calculate the remaining turning movement volumes using mathematical models [11,12]. Although this approach is practical and accurate; the developed models, based on this approach, have many potential shortcomings, as follow. No general models were developed to estimate turning movement volumes for roundabouts with any number of legs. The developed models were based on the assumption that no U-turn movements from and to the same roundabout leg are available. Moreover, the important topic of economizing and simplifying field data collection was not addressed in this approach.

The purpose of this paper is to develop a generalized mathematical model to overcome the above mentioned shortcomings. More specifically this paper aims to:

- Calculate the most difficult turning movements based on counting only out-flows, circulating flows in addition to the easiest turning movements.
- Include the u-turn movements in the calculations, for roundabouts with any number of entries and exits.
- Reduce the number of the detectors or counters in addition to simplify the setting up.
- Present the developed model in an O–D matrix form to be practical and to easily calculate the turning movements for any roundabouts.

The paper is divided into four sections. Following the introduction is a proposed approach of the developed methodology including basic definitions, monitoring area, and equations system. Then, the model development section is presented. Next is a validation of the developed method based on real observation data. Finally, a summary of main conclusions and suggestions for future work is given.

2. Proposed approach

The proposed approach aims at calculating mathematically the most difficult turning movements, based on out-flows, circulating flows, and the easiest turning movements in the roundabout. The criterion used to decide whether the movement is easy or difficult is the size of the area to be monitored to track any vehicle. The following subsections discuss the concept of monitoring area size, equations system and the basic definitions used to develop the proposed model.

2.1. Basic definitions

In this subsection, the used terms in developing the proposed model will be defined.

Out-flow O_i: Total traffic flow leaves leg i (e.g. O_2 means the traffic flow leaves leg/exit 2).
Circulating flow C_i: Total traffic flow circulates in front of leg i (e.g. C_2 means the traffic flow circulates in front of leg 2).
Turning movement M_i^k: Traffic flow leaves the roundabout at leg i and comes from k legs before leg i (e.g. M_3^1 means the traffic flow leaves from leg/exit 3 and comes from leg/entrance 2).

NB: U-turning movement has $k = 0$, right-turn movement has $k = 1$, and left-turn movement has $k = N - 1$, where N is the number of the roundabout legs.

Succeeding turning movement M_{i+j}^k: Traffic flow leaves the roundabout at leg/exit after i by j legs and comes from the leg/entrance before $i + j$ leg by k legs (e.g. M_{2+1}^0, u-turning flow, the traffic flow leaves the roundabout at leg/exit 3 and comes also from leg/entrance 3, another example M_{2+1}^2 is the traffic flow leaves the roundabout at leg/exit 3 and comes from leg/entrance 1).

NB: if $i + j > N$ then $i + j$ should be considered $i + j - N$ (e.g. in the 3-leg roundabout, M_{3+2}^1 means the traffic flow leaves the roundabout at leg/exit 2 and comes from leg/entrance 1).

2.2. Monitoring area

In any roundabout, both out-flows and circulating flows need the smallest monitoring areas to be tracked. Each right-turn needs a monitoring area covering $\frac{1}{N}$ of the roundabout area, where N is the number of roundabout legs. Also, each through movement needs a monitoring area covering $\frac{1}{2}$ of the roundabout area. For left-turn movements, each one needs a monitoring area covering $\frac{N-1}{N}$ of the roundabout area. For U-turns, each movement needs a monitoring area covering $\frac{N-2}{N}$ of the roundabout area. Therefore, the easiest movements are out-flows and circulating flows while the most difficult movements are the left-turns.

In 3-leg roundabouts, the monitoring area size of right-turn and u-turn may be equal, however, u-turn is easier than right-turn due to the possibility of conflict in the case of right-turns. On the other hand, in 4-leg roundabouts, the monitoring area size of through movement and u-turn may be equal; however, u-turn is easier than through movement due to the possibility of conflict in through movements.

2.3. Equations system

The proposed equations system is a determinant system of linear equations having the two difficult turning movements as unknowns and the easiest turning movements in addition to out-flows and circulating flows as inputs. The number of observations should be equal to the number of unknowns, in the determinate equation system. In the 3-leg roundabouts, the unknowns are left-turning, and right-turning movements while the easiest observations are out-flows, circulating flows, and u-turning movements if exist. Similarly, in the 4-leg roundabouts, the unknowns are left-turning, and through movements while the easiest observations are out-flows, circulating flows, right-turning and u-turning movements if existed. Generally, in all roundabouts, the unknowns are the two difficult turning movements and the observations are out-flows and circulating flows, in addition to other easiest movements as the u-turning movements in the 3-leg roundabouts, or the right-turning and u-turning movements in the 4-leg roundabouts.

3. Model development

The proposed model depends on the number of roundabout legs. Therefore, the proposed model will be explained for three-leg, four-leg, and five-leg cases separately. Then a general case for N-leg roundabout is presented. In every case; the easiest and most difficult turning movements are defined, then mathematical equations are used to calculate the most difficult turning movements using the easiest turning movement as well as out-flows and circulating flows. In the three-leg and four-leg roundabout cases, two scenarios are studied concerning the exiting and the absence of u-turns inside the roundabout. In the specific cases, three, four, and five-leg roundabouts, the model is presented in an O–D matrix forms to be practical and user-friendly.

3.1. Three-leg roundabout model

In this case, the unknowns, the most difficult turning movements, are right-turning M_i^1 and left-turning M_i^2, leaving the roundabout at leg i. The required, the easiest, observations are out-flows O_i, circulating flows C_i, and u-turns M_i^0 if exist, as shown in Fig. 1. Table 1 indicates all roundabout turning movements including the unknowns and the required observations (observations are indicated in bold).

There are three equations for out-flows and three others for circulating flows, as follows:

$$\mathbf{O}_i = \boldsymbol{M}_i^0 + M_i^1 + M_i^2 \quad (i = 1, 2, 3) \tag{1}$$

$$\mathbf{C}_i = M_{i+1}^2 + \boldsymbol{M}_{i+1}^0 + \boldsymbol{M}_{i-1}^0 \quad (i = 1, 2, 3) \tag{2}$$

where:

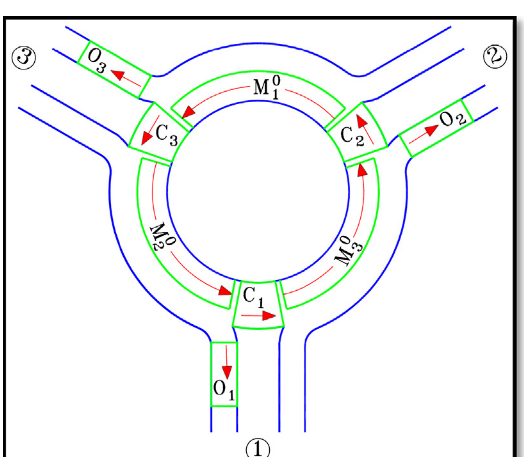

Figure 1 Required observations and their monitoring areas for 3-leg roundabout.

Table 1 Observed and unknown turning movements for 3-leg roundabout.

	To		
From	1	2	3
1	$\boldsymbol{M_1^0}$	M_2^1	M_3^2
2	M_1^2	$\boldsymbol{M_2^0}$	M_3^1
3	M_1^1	M_2^2	$\boldsymbol{M_3^0}$

M_i^2 = left-turning traffic exits the roundabout at leg i and enters from the leg preceding i by two legs;
M_{i+1}^2 = left-turning traffic exits at leg succeeding I and enters from leg preceding to leg i;
$\boldsymbol{M}_{i+1}^0$ = the u-turn for the leg succeeding leg i; and
$\boldsymbol{M}_{i-1}^0$ = the u-turn for the leg preceding leg i.
The unknowns can be determined by solving the above six equations simultaneously, as follows:

3.1.1. Right-turn unknowns

$$M_i^1 = \mathbf{O}_i + \boldsymbol{M}_{i+1}^0 - \mathbf{C}_{i+2} \quad (i = 1, 2, 3) \tag{3}$$

As an example, $M_1^1 = \boldsymbol{O}_1 + \boldsymbol{M}_{1+1}^0 - \boldsymbol{C}_{1+2}$, means that:
Right-turn leaves leg/exit 1 = out-flow for leg 1 + u-turn for leg 3 – circulating flow in front of leg 3.

3.1.2. Left-turn unknowns

$$M_i^2 = \mathbf{C}_{i+2} - \boldsymbol{M}_i^0 - \boldsymbol{M}_{i+1}^0 \quad (i = 1, 2, 3) \tag{4}$$

The three-leg roundabout model, including all turning movements, is summarized and presented in a form of O–D matrix to be practical and to easily calculate the turning movements, as in Table 2.

In the case of no u-turning movements, the matrix will be as shown in Table 3.

3.2. Four-leg roundabout model

In this roundabout, the unknowns, the difficult turning movements, are through movements M_i^2 and left-turning movements M_i^3, leaving leg i. The required traffic counts to be observed are out-flows $\mathbf{O_i}$, circulating flows $\mathbf{C_i}$, right-turning movements $\boldsymbol{M}_i^1$, leaving leg i, assuming that no u-turning movements occur inside the roundabout, as shown in Fig. 2. Table 4 indicates all roundabout turning movements including the unknowns and the required observations (observations are indicated in bold).

Table 2 Turning movements matrix for 3-leg roundabout.

	To		
From	1	2	3
1	$\boldsymbol{M_1^0}$	$\mathbf{O_2} + \boldsymbol{M_3^0} - \mathbf{C_1}$	$\mathbf{C_2} - \boldsymbol{M_3^0} - \boldsymbol{M_1^0}$
2	$\mathbf{C_3} - \boldsymbol{M_1^0} - \boldsymbol{M_2^0}$	$\boldsymbol{M_2^0}$	$\mathbf{O_3} + \boldsymbol{M_1^0} - \mathbf{C_2}$
3	$\mathbf{O_1} + \boldsymbol{M_2^0} - \mathbf{C_3}$	$\mathbf{C_1} - \boldsymbol{M_2^0} - \boldsymbol{M_3^0}$	$\boldsymbol{M_3^0}$

Table 3 Turning movements matrix without u-turns for 3-leg roundabout.

	To		
From	1	2	3
1	0	$\mathbf{O_2}$–$\mathbf{C_1}$	$\mathbf{C_2}$
2	$\mathbf{C_3}$	0	$\mathbf{O_3}$–$\mathbf{C_2}$
3	$\mathbf{O_1}$–$\mathbf{C_3}$	$\mathbf{C_1}$	0

There are four equations for out-flows and four others for circulating flows, as follows:

$$\mathbf{O}_i = \boldsymbol{M}_i^1 + M_i^2 + M_i^3 \quad (i = 1, 2, 3, 4) \tag{5}$$

$$\mathbf{C}_i = M_{i+1}^2 + M_{i+1}^3 + M_{i+2}^3 \quad (i = 1, 2, 3, 4) \tag{6}$$

The unknowns can be determined by solving the above eight equations simultaneously, as follows:

3.2.1. Through movement unknowns

$$M_i^2 = \mathbf{O}_i + \mathbf{O}_{i-1} - \boldsymbol{M}_i^1 - \boldsymbol{M}_{i-1}^1 - \mathbf{C}_{i+2} \quad (i = 1, 2, 3, 4) \tag{7}$$

As an example, $M_1^2 = \mathbf{O_1} + \mathbf{O_{1-1}} - \boldsymbol{M_1^1} - \boldsymbol{M_{1-1}^1} - \mathbf{C_{1+2}}$, means that the through movement leaves exit/leg 1 = out-flow leaves leg 1 + out-flow leaves leg 4 – right -turn leaves to leg 1 – right-turn leaves leg 4 – circulating flow in front of leg 3.

3.2.2. Left-turn unknowns

$$M_i^3 = \mathbf{C}_{i+2} - \mathbf{O}_{i-1} + \boldsymbol{M}_{i-1}^1 \quad (i = 1, 2, 3, 4) \tag{8}$$

Table 5 summarizes the final solution.

If any u-turning movement occurs, its value should be observed and subtracted from all out-flows and circulating flows containing it. In other words, if u-turn of leg 1 occurs inside the roundabout, then it should be observed using the monitoring area indicated in Fig. 3 and then it should be subtracted from out-flow leaving leg O_1, circulating flows; C_2, C_3,

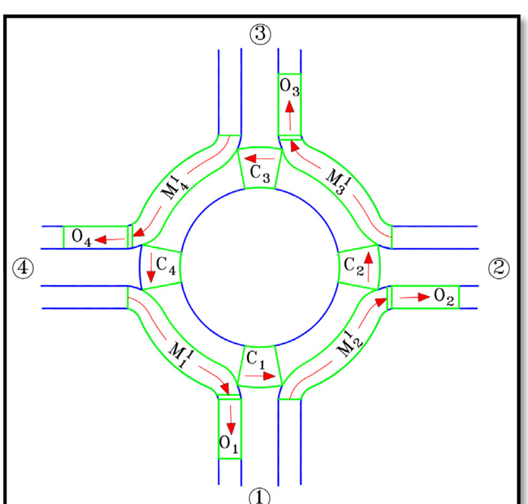

Figure 2 Required observations and their monitoring areas for 4-leg roundabout without u-turn movements.

Table 4 Turning movements for 4-leg roundabout without u-turns.

	To			
From	1	2	3	4
1	0	$\boldsymbol{M}_2^1$	M_3^2	M_4^3
2	M_1^3	0	$\boldsymbol{M}_3^1$	M_4^2
3	M_1^2	M_2^3	0	$\boldsymbol{M}_4^1$
4	$\boldsymbol{M}_1^1$	M_2^2	M_3^3	0

and C_4. Similarly, if all u-turning movements occur inside the roundabout, then the unknowns will be as follows:

3.2.3. Through movement unknowns with u-turns

$$M_i^2 = \mathbf{O}_i + \mathbf{O}_{i-1} - \boldsymbol{M}_i^1 - \boldsymbol{M}_{i-1}^1 - \mathbf{C}_{i+2} + M_{i+1}^0 \quad (i = 1, 2, 3, 4) \tag{9}$$

3.2.4. Left-turn unknowns with u-turns

$$M_i^3 = \mathbf{C}_{i+2} - \mathbf{O}_{i-1} + \boldsymbol{M}_{i-1}^1 - M_i^0 - M_{i+1}^0 \quad (i = 1, 2, 3, 4) \tag{10}$$

The turning movements, in the case of u-turns, are indicated in Table 6.

3.3. Five-leg roundabout model

In this case, the unknowns are turning movements M_i^4 and M_i^3 while the observations are out-flows $\mathbf{O_i}$, circulating flows $\mathbf{C_i}$, and turning movements $\boldsymbol{M}_i^1$ and $\boldsymbol{M}_i^2$ in addition to u-turn movements $\boldsymbol{M}_i^0$ if exist. Table 7 indicates the unknown turning movements and the required observations (observations are indicated in bold).

There are five equations for out-flows and other five equations for circulating flows.

The unknowns can be determined by solving the 10 equations simultaneously and the results will be as follows:

3.3.1. The 3rd turning movement unknowns

$$M_i^3 = \mathbf{O}_i + \mathbf{O}_{i-1} + \mathbf{O}_{i-2} - \mathbf{C}_{i+2} + \boldsymbol{M}_{i+1}^0 - \boldsymbol{M}_i^1 - \boldsymbol{M}_{i+3}^1 - \boldsymbol{M}_{i+4}^1 - \boldsymbol{M}_i^2 - \boldsymbol{M}_{i+4}^2 \quad (i = 1, 2, 3, 4, 5) \tag{11}$$

3.3.2. The 4th turning movement unknowns

$$M_i^4 = \mathbf{C}_{i+2} - \mathbf{O}_{i-1} - \mathbf{O}_{i-2} - \boldsymbol{M}_i^0 - \boldsymbol{M}_{i+1}^0 + \boldsymbol{M}_{i+3}^1 + \boldsymbol{M}_{i+4}^1 + \boldsymbol{M}_{i+4}^2 \quad (i = 1, 2, 3, 4, 5) \tag{12}$$

Table 8 summarizes the final solution.

3.4. N-leg roundabout model

The general model for N-leg roundabout has two linear equations; the first equation is for the left-turning movements M_i^{N-1} and the second equation is for the turning movements M_i^{N-2}. Both equations are based on out-flows O_i, circulating flows C_i in addition to the easiest turning movements M_i^{N-3},

Table 5 Turning movements matrix without u-turns for 4-leg roundabout.

From	To 1	2	3	4
1	0	M_2^1	$\mathbf{O}_3 + \mathbf{O}_2 - M_3^1 - M_2^1 - \mathbf{C}_1$	$\mathbf{C}_2 - \mathbf{O}_3 + M_3^1$
2	$\mathbf{C}_3 - \mathbf{O}_4 + M_4^1$	0	M_3^1	$\mathbf{O}_4 + \mathbf{O}_3 - M_4^1 - M_3^1 - \mathbf{C}_2$
3	$\mathbf{O}_1 + \mathbf{O}_4 - M_1^1 - M_4^1 - \mathbf{C}_3$	$\mathbf{C}_4 - \mathbf{O}_1 + M_1^1$	0	M_4^1
4	M_1^1	$\mathbf{O}_2 + \mathbf{O}_1 - M_2^1 - M_1^1 - \mathbf{C}_4$	$\mathbf{C}_1 - \mathbf{O}_2 + M_2^1$	0

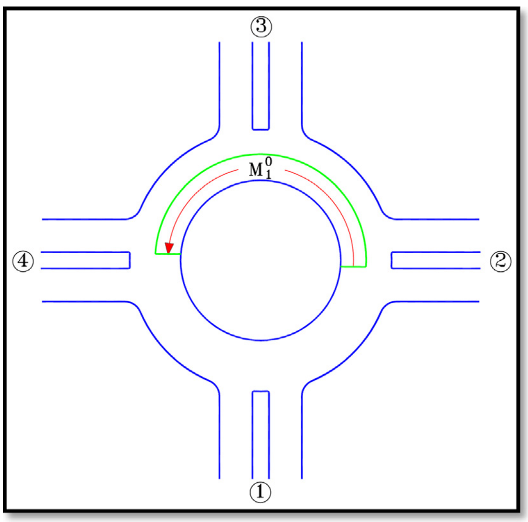

Figure 3 U-turn and its monitoring areas for 4-leg roundabout.

$M_i^{N-4}, \ldots, M_i^0$. Table 9 indicates the unknown turning movements and the required observations (observations are indicated in bold).

There are N equations for out-flows O_i and N others for circulating flows C_i. The unknowns can be determined by solving the $2N$ equations simultaneously and the results are as follows:

3.4.1. The $(N-1)$th turning movement unknowns

$$M_i^{N-1} = \mathbf{C}_{i+2} - \sum_{k=1}^{k=N-3} \mathbf{O}_{i-k} - \sum_{j=1}^{j=N-1} M_{i+j+2}^0 + \sum_{j=3}^{j=N-1}\sum_{k=0}^{k=N-3} M_{i+j}^k - \sum_{j=1}^{j=N-4}\sum_{k=j+1}^{k=N-3} M_{i+j+2}^k \quad (i = 1, 2, \ldots, N) \tag{13}$$

Table 7 Turning movements for 5-leg roundabout.

From	To 1	2	3	4	5
1	$\mathbf{M}_1^0$	$\mathbf{M}_2^1$	$\mathbf{M}_3^2$	M_4^3	M_5^4
2	M_1^4	$\mathbf{M}_2^0$	$\mathbf{M}_3^1$	$\mathbf{M}_4^2$	M_5^3
3	M_1^3	M_2^4	$\mathbf{M}_3^0$	$\mathbf{M}_4^1$	$\mathbf{M}_5^2$
4	$\mathbf{M}_1^2$	M_2^3	M_3^4	$\mathbf{M}_4^0$	$\mathbf{M}_5^1$
5	$\mathbf{M}_1^1$	$\mathbf{M}_2^2$	M_3^3	M_4^4	$\mathbf{M}_5^0$

3.4.2. $N-2$th turning movement unknowns

$$M_i^{N-2} = \sum_{k=0}^{k=N-3} \mathbf{O}_{i-k} - C_{i+2} + \sum_{j=1}^{j=N-1} M_{i+j+2}^0 - \sum_{j=3}^{j=N}\sum_{k=0}^{k=N-3} M_{i+j}^k + \sum_{j=1}^{j=N-4}\sum_{k=j+1}^{k=N-3} M_{i+j+2}^k \quad (i = 1, 2, \ldots, N) \tag{14}$$

4. Model validation

In order to check the validity of the developed mathematical model, a web-based one hour video traffic count at 4-leg roundabout was used [13] as indicated in Fig. 4.

To extract the required data, two separate computer-based traffic surveys were performed. The first survey is to obtain the traffic count for all roundabout turning movements using the direct manual count to be the reference data. The second survey is to obtain only out-flows $\mathbf{O_i}$, circulating flows $\mathbf{C_i}$, and right-turning movements M_i^1, as indicated in Table 10, to be used as input in the 4-leg roundabout model. In both surveys, careful consideration was given to obtain the exact count for each required movement. Therefore three independent surveyors were assigned to count each movement. Cross validation among the data collected for each movement, by the three surveyors, in each survey, was carried out. Any inconsistency was revised and corrected.

Table 6 Turning movements matrix with u-turns for 4-leg roundabout.

From	To 1	2	3	4
1	M_1^0	M_2^1	$\mathbf{O}_3 + \mathbf{O}_2 - M_3^1 - M_2^1 - \mathbf{C}_1 + M_4^0$	$\mathbf{C}_2 - \mathbf{O}_3 + M_3^1 - M_4^0 - M_1^0$
2	$\mathbf{C}_3 - \mathbf{O}_4 + M_4^1 - M_1^0 - M_2^0$	M_2^0	M_3^1	$\mathbf{O}_4 + \mathbf{O}_3 - M_4^1 - M_3^1 - \mathbf{C}_2 + M_1^0$
3	$\mathbf{O}_1 + \mathbf{O}_4 - M_1^1 - M_4^1 - \mathbf{C}_3 + M_2^0$	$\mathbf{C}_4 - \mathbf{O}_1 + M_1^1 - M_2^0 - M_3^0$	M_3^0	M_4^1
4	M_1^1	$\mathbf{O}_2 + \mathbf{O}_1 - M_2^1 - M_1^1 - \mathbf{C}_4 + M_3^0$	$\mathbf{C}_1 - \mathbf{O}_2 + M_2^1 - M_3^0 - M_4^0$	M_4^0

Table 8 Turning movements matrix for 5-leg roundabout.

From	To 1	2	3	4	5
1	M_1^0	M_2^1	M_3^2	$\mathbf{O}_4+\mathbf{O}_3+\mathbf{O}_2-\mathbf{C}_1+M_5^0-M_4^1$ $-M_2^1-M_3^1-M_4^2-M_3^2$	$\mathbf{C}_2-\mathbf{O}_4-\mathbf{O}_3-M_5^0-M_1^0+M_3^1$ $+M_4^1+M_4^2$
2	$\mathbf{C}_3-\mathbf{O}_5-\mathbf{O}_4-M_1^0-M_2^0$ $+M_4^1+M_5^1+M_5^2$	M_2^0	M_3^1	M_4^2	$\mathbf{O}_5+\mathbf{O}_4+\mathbf{O}_3-\mathbf{C}_2+M_1^0-M_5^1$ $-M_3^1-M_4^1-M_5^2-M_4^2$
3	$\mathbf{O}_1+\mathbf{O}_5+\mathbf{O}_4-\mathbf{C}_3+M_2^0$ $-M_1^1-M_4^1-M_5^1-M_1^2-M_5^2$	$\mathbf{C}_4-\mathbf{O}_1-\mathbf{O}_5-M_2^0-M_3^0$ $+M_5^1+M_1^1+M_1^2$	M_3^0	M_4^1	M_5^2
4	M_1^2	$\mathbf{O}_2+\mathbf{O}_1+\mathbf{O}_5-\mathbf{C}_4+M_3^0$ $-M_2^1-M_5^1-M_1^1-M_2^2-M_1^2$	$\mathbf{C}_5-\mathbf{O}_2-\mathbf{O}_1-M_3^0-M_4^0$ $+M_1^1+M_2^1+M_2^2$	M_4^0	M_5^1
5	M_1^1	M_2^2	$\mathbf{O}_3+\mathbf{O}_2+\mathbf{O}_1-\mathbf{C}_5+M_4^0$ $-Ms_3^1-M_1^1-M_2^1-M_3^2-M_2^2$	$\mathbf{C}_1-\mathbf{O}_3-\mathbf{O}_2-M_4^0-M_5^0$ $+M_2^1+M_3^1+M_3^2$	M_5^0

Table 9 Turning movements for N-leg roundabout.

From	To 1	2	3	...	$N-1$	N
1	M_1^0	M_2^1	M_3^2	...	M_{N-1}^{N-2}	M_N^{N-1}
2	M_1^{N-1}	M_2^0	M_3^1	...	M_{N-1}^{N-3}	M_N^{N-2}
3	M_1^{N-2}	M_2^{N-1}	M_3^0	...	M_{N-1}^{N-4}	M_N^{N-3}
...	...	...	...	...	...	...
$N-1$	M_1^2	M_2^3	M_3^4	...	M_{N-1}^0	M_N^1
N	M_1^1	M_2^2	M_3^3	...	M_{N-1}^{N-1}	M_N^0

Figure 4 The used roundabout in model validation.

Table 10 The observations in the second survey.

Leg (i)	1	2	3	4
$\mathbf{O}_i$	635	229	818	333
$\mathbf{C}_i$	336	872	229	570
M_i^1	102	61	40	142

Table 11 The reference and calculated turning movements.

From	To 1	2	3	4
1	**0 (0)**	**61 (61)**	610 (610)	94 (99)
2	38 (32)	**0 (0)**	**40 (40)**	98 (93)
3	496 (495)	37 (39)	**0 (0)**	**142 (142)**
4	**102 (102)**	131 (134)	168 (165)	**0 (0)**

A 4-leg roundabout model, as in Table 5, was used to calculate the full turning movements, based on the data collected in the 2nd survey. Table 11 shows the reference turning movements, as in parentheses, as well as the calculated count using the proposed model. Comparing both counts indicates an agreement between reference and calculated turning movements. Such results reflect the accuracy of the developed models to be used for calculation. However, other validations should be conducted using more roundabouts types, upon data availability.

It is worth mentioning that the time consumed to extract the reference turning movements was 12 h in the first survey; one hour for every movement. While the time in the second survey was 8 h; one hour for right-turn for every leg and one hour for both out-flow and circulating flow at every leg. These results mean that the proposed model achieves accurate counts with less cost of man-hours in the site for the case of manual survey or in the office for the case of using video camera.

5. Conclusions and recommendations for future work

Tracking vehicles through roundabouts is not a straightforward task. There is, then, a need to find a suitable and practical method to acquire turning movement volumes at roundabouts. The primary objective of this paper is to develop a generalized mathematical model that calculates the turning movement volumes for any roundabout with any number of legs. In this paper, a simplified model was developed and presented to calculate mathematically the most difficult turning movements based on surveying out-flows, circulating flows, and the easiest turning movements. The criterion used to decide whether the movement is easy or difficult is based on the size of the area to be monitored to track any vehicle movement. The developed model considers the u-turn movements in the calculations for roundabouts with any number of entries and exits. It seems that no satisfactory model that considers u-turns was found by previous researchers. The model has cost-effective and practical use in reducing the number of detectors or counters in addition to simplify their setting up (whether video, automatic or manual techniques are in use). Based on the model, the efforts of the field counts will be concentrated on certain sections/movements of the roundabout without the need to survey all movements of a roundabout. The reduction in the number of detectors could be due to site limitations or other reasons. Moreover, the model was presented in a form of O–D matrix to be practical and to easily calculate the turning movements. Furthermore, the model was validated against reference traffic count data and the results were found to be satisfactory. The validation example ensured that the proposed model can significantly reduce the cost of man-hours needed in site or in office. On the other hand, the proposed model is useful to overcome the tracking problem in the large roundabout or in the case of vision obstacles.

Although, the developed model can be used for roundabouts with U-turn movements and under any traffic condition, the validation is based on one roundabout without any U-turn movements and carrying relatively moderate traffic volumes. However, it is hoped, upon data availability, to validate the developed model using more cases with U-turn movements and under heavy traffic volumes.

Finally, it is recommended to carry out more investigations to examine the developed models using comprehensive field data for roundabouts with different legs.

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